Comparative analysis of the mathematical models of the dynamics of the coronavirus COVID-19 epidemic development in the different countries

P. V. Khrapov, A. A. Loginova

Abstract—**In this work, mathematical modelling of the dynamics of coronavirus COVID-19 is performed for the following countries: the USA, Germany, the UK and the Russian Federation. Cases of COVID-19 virus have been detected in nearly 200 countries. On February 11, 2020, the World Health Organization decided to officially name the virus SARSCoV-2, and the disease caused by this virus - COVID-19 On March 11, WHO Director-General Tedros Gebreyesus announced that the spread of coronavirus infection COVID-19 "could be described as a pandemic." Viral incubation period of this virus is quite long, it ranges from 1 to 14 days. The danger of a new disease also lies in the fact that it is easily confused with a common cold or flu. Therefore, the spread of coronavirus COVID-19 is a serious threat for international health and economics. A mathematical description of the dynamics of virus allows to study the nature of the disease thoughtfully, to analyze statistical and model data, to make hypotheses concerning the future dynamics of coronavirus and to evaluate the effectiveness of the measures undertaken. For mathematical modelling of coronavirus COVID-19, the authors use a modified system of differential equations constructed according to the SIR compartmental model. The optimal values of the model parameters, that describe the statistical data precisely, were found. The analysis of the current situation of the COVID-19 coronavirus epidemic in each considering country was made, which led to the efficiency mark of the existing measures to struggle against the virus in the USA, Germany, the UK and the Russian Federation.**

Keywords—COVID-19; coronavirus; mathematical modelling; epidemiological modelling; SIR model.

I. Introduction

Each new infectious disease is a serious challenge for modern medicine. Rapid environmental changes, global environmental problems – this is only a small part of the factors influencing the emergence of new pathogens [1]. The 2020th year will be remembered for a long time thanks to the emergence of a new coronavirus – COVID-19. Nowadays more than 2 million infections were recorded, of which the number of deaths is more than 150 thousand people. The coronavirus COVID-19 was founded in almost every country in the world. The most affected by the pandemic are such regions as mainland China, Europe (Italy, Spain, Germany, France, the UK), the USA, Iran, and Turkey.

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In the general case, Coronaviridae is a family of viruses consisting of pathogens in both humans and animals [2]. The family so far includes 40 different types of viruses. The most famous representatives of this group are the SARS-CoV (the causative agent of SARS, the first case of which was registered in 2002) and the MERS-CoV (the causative agent of the Middle Eastern respiratory syndrome, the outbreak of which occurred in 2015) [3,4]. The name is associated with the structure of the virus: under an electron microscope their shape resembles a corona. The severity of the disease in people infected with various viruses of the family is different and varies from a mild illness that resembles a mild cold to a disease with severe clinical symptoms. Fatal outcomes are also possible.

COVID-19 has become the third best known coronavirus [5]. It was first discovered on December 31, 2019 in Wuhan in Hubei Province, China. After that, the spread of the disease was recorded in other provinces of China, as well as in other countries of the world. The virus is transmitted from person to person, most likely through contact with discharge from the respiratory tract. Washing your hands regularly is the best way to individually protect yourself from the virus. This measure eliminates possible viral contamination of the hands and avoids infection if a person touches his eyes, mouth or nose. Symptoms of the disease caused by the COVID-19 coronavirus are similar to flu symptoms: fever, dyspnea, and labored breathing. Less common headaches, heart palpitations, dizziness, abdominal pain, nausea and vomiting [6]. Moreover, during a clinical examination it is not possible to distinguish infection with a new coronavirus from other respiratory diseases. With a viral infection of COVID-19, can occur such complications as sinusitis pneumonia, bronchitis, acute respiratory failure, pulmonary edema, sepsis and infectious toxic shock can occur. Diagnosis of the disease is made only through laboratory analysis of discharge from the respiratory tract. For most people the disease ends with recovery, and specific treatment measures are not required. Particularly the elderly people are at risk, especially those with chronic diseases. The mortality rate among infected under the age of 40 years is 0.2%, at the age of 70-79 years - 8%, and from 80 years old - about 14.8%. Obviously, coronavirus infection with COVID-19 poses a great threat to the elderly people. There is no vaccine against the virus nowadays.

Developing a vaccine is being carried out by teams all around the world, but no effective vaccine is expected in the near future [7,8]. Nevertheless, traditional medicine can help to improve well-being and alleviate the symptoms of COVID-19.

Currently, several scientists are trying to predict the behavior of the coronavirus COVID-19 [9, 10, 11, 12, 13, 14]. At this moment due to lack of statistical data, all works are preliminary [15]. The SIR model is one of them. SIR model is widely used in mathematical epidemiology [16]. For example, it is used for modelling Ebola epidemic [16,17] and HIV/ AIDS epidemic [18,19,20]. Through research of the spread of COVID-19, scientists can come up with useful insights that will help in combating the pandemic in the future.

II. Mathematical modelling of the dynamics of the coronavirus COVID-19 epidemic development

In this work, mathematical model for the coronavirus COVID-19 epidemic development is performed on the basis of the epidemiological SIR model. The same model is used for mathematical modelling of the dynamics of the coronavirus COVID-19 epidemic development in China [22].

The epidemiological SIR model is defined by the system of differential equations Let N be the population of the risk group. Morbidity is proportional to the number of contacts between people of the susceptible $S(t)$ and the infected $I(t)$. Rate at which people leave the infected group $I(t)$, i.e. recover or die, is proportional to the population of this group. Moreover, the severity of coronavirus COVID-19 disease depends on the characteristics of an individual organism. Both cases of complete recovery and cases of death are known. In this case, the rick group can be divided into four group in each compartment at a particular time t:

- $S(t)$ (susceptible) – healthy individuals, who are at the risk group and who can catch the infection;
- $I(t)$ (infected) – infected individuals, who are carries of infection;
- $\tilde{R}(t)$ (recovered) – recovered individuals and individuals who are immune to this disease;
- $D(t)$ (dead) – dead individuals.

For the above groups the following condition is fulfilled:

$$N = S(t) + I(t) + \tilde{R}(t) + D(t).$$

Moreover, it is worth considering that there are certain periods between the onset of the disease, its diagnosis, recovery or death of the patient. Based on the above considerations, the modified system of differential equations can be written in the following form:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\alpha S(t) I(t-\tau_1), \\ \dfrac{dI(t)}{dt} = \alpha S(t) I(t-\tau_1) - \beta I(t-\tau_2) - \gamma I(t-\tau_3), \\ \dfrac{d\tilde{R}(t)}{dt} = \beta I(t-\tau_2), \\ \dfrac{dD(t)}{dt} = \gamma I(t-\tau_3), \end{cases} \quad (1)$$

where

α - the average infection rate of an individual,

β - the average recovery rate of an individual,

γ - the average rate of death of an individual,

τ_1 - the average time of the incubation period

τ_2 - the average recovery time of an individual,

τ_3 - the average time of death of an individual.

By setting the values of α, β, γ, τ_1, τ_2, τ_3, the initial values of N, I, $\tilde{R}$, D, the dynamics of the disease can be calculated. The differential equations system with initial conditions (1) is solved using numerical methods. Let us use the system of differential system (1) for mathematical modelling of the dynamics of the coronavirus COVID-19 epidemic development in the USA, Germany, the UK and the Russian Federation.

A. The USA

The first case of coronavirus infection in the USA was firstly recorded in January 2020. On March 26, the USA have already come to the first place in the world in the number of infected. Nowadays the number of infected has almost reached one million. Most likely, the reason for the outbreak was that at the beginning of the pandemic, the United States decided to follow the path of rapid immunization, i.e. obtaining collective immunity due to a greater number of patients. The decision not to impose total quarantine was due to concerns about the economy. As a result, quarantine measures were not started on time, which led to this outcome. According to experts, the features of the American health care system has also influenced to the deteriorating situation. Thus, the disadvantaged groups without medical insurance and comfortable living conditions are most susceptible to the coronavirus infection. In the considered model, the size of the risk group $N = 890000$ people, time shifts $\tau_1 = 5.1$, $\tau_2 = 14$, $\tau_3 = 9$ [WHO], parameters $\alpha = 0.292$, $\beta = 0.0205$, $\gamma = 0.008$ (fig. 1). We can see a good agreement of the statistical data with the model curves. Based on the statistical and model curves, we can conclude that the epidemic peak in the USA will be at the end of April. Moreover, the number of dead individuals can be more than 100 thousand people.

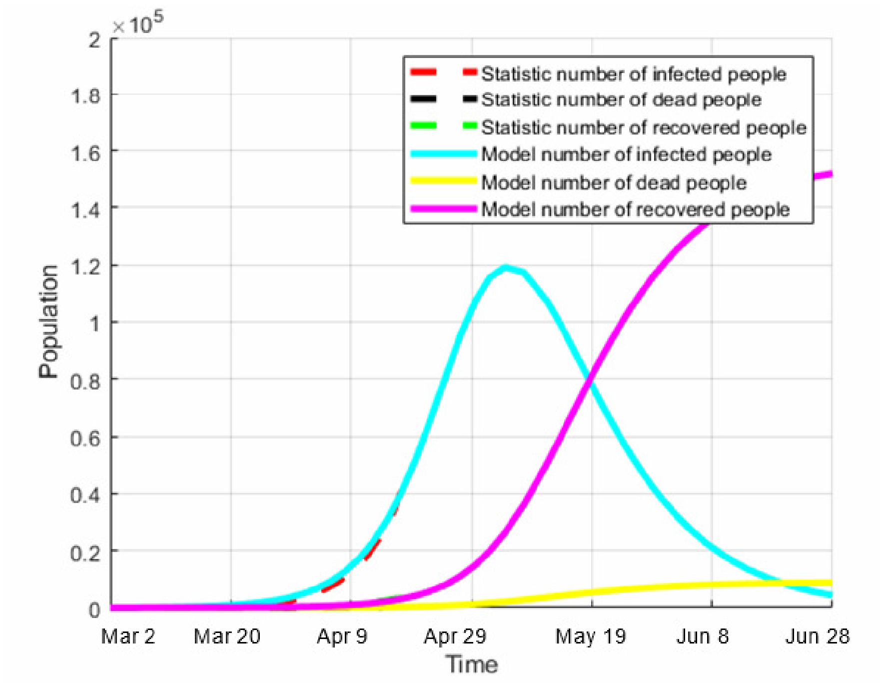

Fig. 1. Dynamics of coronavirus COVID-19 spreading in the USA.

B. Germany

In the present time Germany is on the fifth place in the world in the number of infected. The first case of infection has recorded on January 27. Then, on March 17, the Robert Koch Institute increased the epidemiological risk for the country from medium to high level. The distinctive feature of the spread of coronavirus COVID-19 in Germany is the small number of dead: despite the fact that the number of infected is already more than 100 thousand people, the number of dead is only 3 thousand. This can be described with good equipment of German hospitals and clinics. Let us choose the size of the risk group $N = 150000$ people, time shifts $\tau_1 = 5.1$, $\tau_2 = 10$, $\tau_3 = 7$ [WHO], parameters $\alpha = 0.257$, $\beta = 0.053$, $\gamma = 0.032$ (fig. 2). We can see a good correspondence of model data curve to the statistical data curve. Based on the model, we can assume that the coronavirus COVID-19 epidemic development is on the decline in Germany. Moreover, the number of dead individuals will not reach 10 thousand people. Thus, we can conclude that the national policy is effective in Germany.

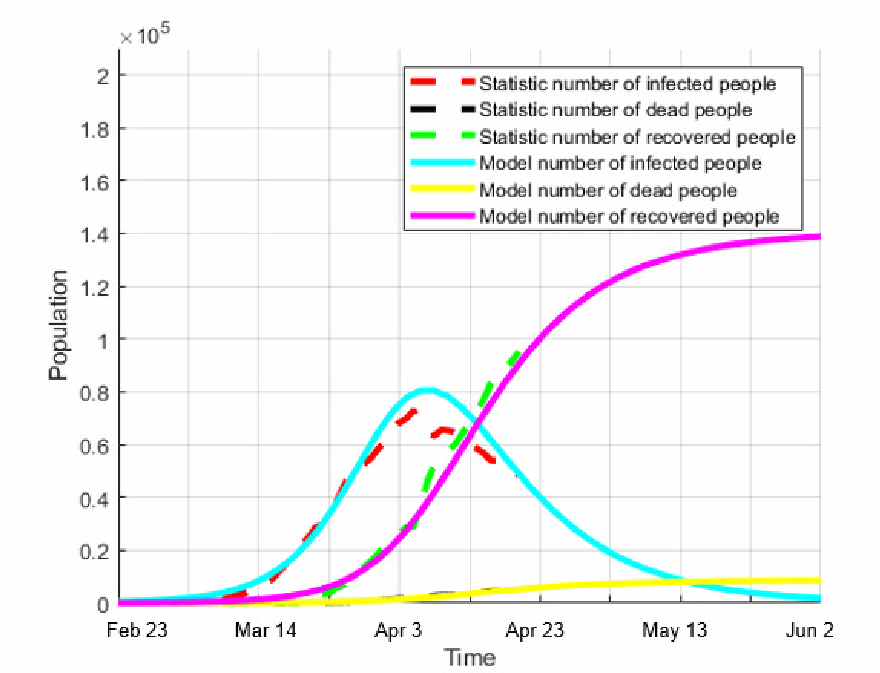

Fig. 2. Dynamics of coronavirus COVID-19 spreading in Germany.

C. The UK

As Germany, the UK is on the list of countries most affected by coronavirus in Europe. In the present time the number of infected individuals is more than 100 thousand people, the number of dead is more than 15 thousand people, but there are almost no recovered. Firstly, the UK government kept the country open without any restrictions. Health care system in the UK is similar to the Italy health care system, and this led to the current situation. In the considered model, the size of the risk group $N = 160000$ people, time shifts $\tau_1 = 5.1$, $\tau_2 = 10$, $\tau_3 = 7$ [WHO], parameters $\alpha = 0.283$, $\beta = 0.02$, $\gamma = 0.021$ (fig. 3). A good correspondence of modelled data to the statistical data can be observed. Due to the lack of statistics on the number of cases, model curves were constructed only for infected and dead. We can see that the decline of coronavirus infection development will start in the second part of April, and the number of dead individuals can be about 40 thousand people.

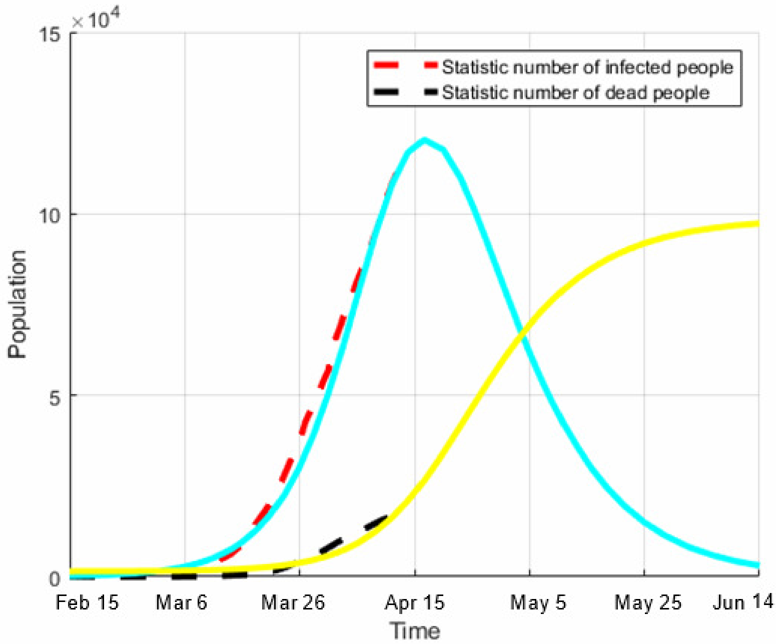

Fig. 3. Dynamics of coronavirus COVID-19 spreading in the UK.

D. The Russian Federation

The rapid spread of coronavirus infection COVID-19 was also recorded in the Russian Federation. Despite the relatively low number of infected, COVID-19 was recorded in all regions of Russia. As in other countries, the set of measures was taken to prevent the spread of infection, including the establishment of non-working days. It seems that the timely taken measures were able to prevent a serious outbreak of infection. Authors take the size of the risk group $N = 165000$ people, time shifts $\tau_1 = 5.1$, $\tau_2 = 10$, $\tau_3 = 7$ [WHO], parameters $\alpha = 0.305$, $\beta = 0.035$, $\gamma = 0.002$ (fig. 4). We can see a good agreement of the statistical data with the model curves. Based on the statistical and model curves, we can assume that the epidemic peaks will be at the beginning of May, after which the epidemic declined. The number of dead individuals will be less than 10 thousand people. This fact shows the effectiveness of state action.

III. CONCLUSION

The beginning of 2020 was overshadowed by the outbreak of the pandemic of the new coronavirus COVID-19. This new type of virus spreads extremely quickly [23]. Every day tens of thousands of new infection cases are recorded worldwide. The world is in a state of uncertainty and does not know what will happen tomorrow. In addition to obvious threats to international public health, this disease seriously affects the international economies. Now it is very important to investigate this pandemic as much as possible [24, 25].

This work presents a mathematical model of the spread of the COVID-19 coronavirus epidemic in the USA, Germany, the UK and the Russian Federation. As the result, in the USA the peak of the coronavirus infection is expected at the end of April, and the number of dead can be more than 100 thousand people. In Germany the forecast is more optimistic: the epidemic the epidemic is already declining, the number of dead should not be more than 10 thousand people. In the UK, like in the USA, the coronavirus infection will soon decline, namely at the end of April. The number of dead individuals, probably, can be about 40 thousand people. In Russia the epidemic peak can be expected at the beginning of May. Moreover, the number of dead should not be large – less than 10 thousand people.

Based on the high adequacy of the presented mathematical model, the authors suggest that its use will help in the study of the spread of COVID-19 coronavirus infection in the different countries. Due to the considering examples, it is in our power to limit the spread of disease outbreaks and stop the transmission of infection.

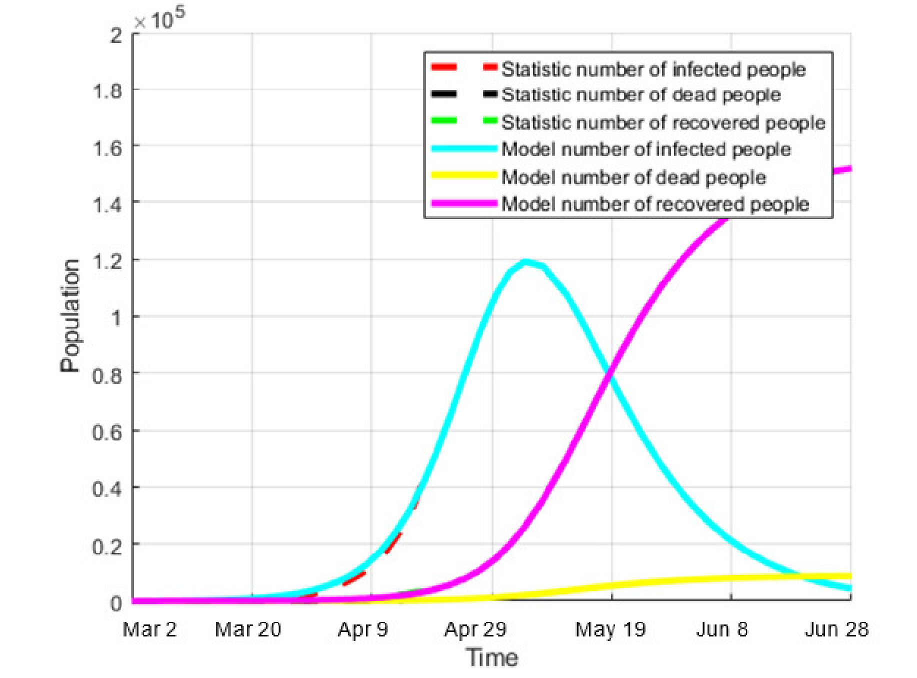

Fig. 4. Dynamics of coronavirus COVID-19 spreading in Russian Federation.

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APENDIX

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