

# Spectral Inversion in Estimation of Change in the Dominant Frequency of the Wave Field

F.V. Krasnov, A.V. Butorin

**Abstract**— This article proposes a method of estimating the dominant frequency of a wave field along a seismic trace, developed on the basis of the spectral inversion method. The basis of the proposed algorithm is the estimation of the dominant frequency from the results of approximation of the seismic trace by wavelets from the given library. The study described was to determine the basic requirements for the formation of the wavelet library, as well as to assess the sensitivity and stability of the proposed algorithm under conditions of different contrast values at the dominant frequency, and the level of additive random interference.

The practical value of the proposed approach is to obtain additional characteristics of the observed wave field for the purpose of geological interpretation of seismic data. Among the key factors influencing the changes in the dominant frequency are: the absorbing characteristics of the medium, depending on the structure of the object under study, the interference effects that lead to a change in the frequency of the total pulse, and the influence of fracturing.

As a result of the research, practical recommendations for the implementation of the proposed algorithm are provided, as well as the sensitivity and noise immunity of the method.

**Key words**— signal model, convolution, matching pursuit, dictionary learning.

## I. INTRODUCTION

The object of the study is a method of restoring the value of the dominant frequency of the wave field along the seismic trace. At the heart of the method lays the spectral inversion algorithm, which consists in a flattened approximation of a seismic trace by a set of wavelets from a predefined library.

The justification for the development of the method for reconstructing the dominant frequency is the need to obtain additional information on the geological structure from the seismic wave field. This task is relevant in the context of the complex structure of geological objects, as well as the lack of geological information.

Information on the frequency composition of the wave field is widely used in modern algorithms for interpretation of seismic data. A significant number of publications shows the practical applicability of spectral characteristics in solving problems of mapping geological objects and

predicting their reservoir properties [7, 8, 11]. In addition, some authors note the correspondence of gas-saturated intervals to regions of dominant frequency decreases [6]. In the light of this fact, obtaining additional characteristics of the objects under study in a wave field increases the accuracy of geological models.

Most algorithms for interpreting seismic data use standard characteristics used in the magnitude of the spectral components, which can be obtained using different methods of spectral decomposition. In the future, the magnitude of the spectral components can be used as an independent parameter of the wave field, or in combination with other components within the framework of the RGB-visualization technique [1, 2]. In the framework of this study, the authors propose a new technique for interpreting the results of spectral inversion, which consists in the pathwise obtaining of the curve of the dominant frequency of the wave field.

The proposed methodology is implemented as a software module, which operation was tested on simple models of the seismic trace. The article describes the study of several practical questions: the possibility of using the proposed algorithm for solving the problem is demonstrated, the problems of the formation of the wavelet dictionary are considered, the stability of the result is investigated depending on the contrast value and the presence of the noise component. The performed research allows defining the applicability limits of the proposed algorithm and the optimal approach to its implementation.

## II. FORMULATION OF THE PROBLEM

The amplitude-frequency characteristic of the signal received at the surface depends on a number of factors:

- The conditions of excitation and registration - mainly depend on the structure of the near-surface part of the medium, as well as technical parameters. Based on this specifics, these factors are not further considered;
- Absorbing properties of the medium - determined by the lithological composition of the deposits, fluid in the pore space, fracturing and other properties. This factor can be described by the "quality factor" of the medium;
- Tuning-effect - this factor is associated with the interference interaction of the reflected waves, which affects the "apparent" dominant value of the total signal.

The influence of these factors is described in more detail in [10]. From the point of view of geological modeling the key two are the last two factors connected with the structure of the target geological objects. In this case, it should be noted that in real geological environments, both these factors affect the frequency composition of the wave field.

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Thus, information on the change in the dominant value of the frequency of the wave field can be used to solve a certain range of practical problems: the study of changes in the structure of the target strata, their fracturing and even saturation (which is relevant for gas deposits).

These prerequisites pose the urgency of developing a method for estimating the change in the dominant frequency, which is stable to the presence of a noise component, and capable of determining the contrasts of the dominant frequency at  $n * 1$  Hz.

### III. METHODS OF SPECTRAL INVERSION

A convolution model is used in the formulation of the spectral inversion problem, which describes the seismic trace ( $s(t)$ ) as a result of convolution of the trace of the reflection coefficients ( $r(t)$ ) with some wavelet ( $w(t)$ ):

$$s(t) = w(t) * r(t) + n(t)$$

Based on this representation of the seismic trace, the concept of a multi-wavelet convolutional model is introduced within the framework of spectral inversion:

$$s(t) = \sum_k^K [w(t, k) * r(t, k)] + n(t)$$

The index  $k$  corresponds to a specific wavelet from the library  $D$ , to which corresponds a specific trace of the reflection coefficients ( $r(t, k)$ ). Thus, the seismic trace can be described by a combination of a set of wavelets, each of which corresponds to a wavelet-dependent trace of the reflection coefficients. Such a model is a physical abstraction, however, if wavelets in the library differ from each other by a dominant frequency, such a representation allows a detailed reconstruction of the wave-field spectrum. The presented convolutional model can be expressed in a matrix form, where  $D$  is the wavelet matrix (wavelet library),  $m$  is the matrix of the corresponding wavelet-dependent reflection coefficients,  $n$  is the additive noise:

$$s = (w_1 \dots w_k) \begin{pmatrix} r_1 \\ r_k \end{pmatrix} + n = Dm + n$$

Such task formulation as search for wavelet-dependent reflection coefficients for a given wavelet library, is the inverse problem of geophysics, is posed incorrectly. First, the given problem does not have a unique solution, and, secondly, the solution is unstable, as small changes in the argument can lead to significant variations of the function. The linear regression problem is solved using the method of least squares:

$$J = \|s - Dm\|^2 \rightarrow \min$$

In order to avoid retraining linear regression, it is necessary to impose restrictions on the variability of the decision rule, such an approach is based on the regularization method. From the Bayesian point of view, the regularization corresponds to the addition of some a priori distributions to the parameters of the model. This allows us to consider the problem of finding a solution as an optimization of the regularized functional [5]:

$$\|s - Dm\|^2 + \lambda \|m\|_{L_p}^p \rightarrow \min, \lambda \geq 0$$

Where  $\|m\|_{L_p}$  is  $L_p$  norm of the form  $\sqrt[p]{\sum |m_j|^p}$  imposing a restriction on the result of the solution. In this connection, we usually consider several types of regularization [3]:

- $L_0$ - regularization - specifies the number of weights other than zero, in which case the solution has the property of rarefaction;
- $L_1$ - regularization - sets the total value of the weights within the solution, in this case the solution is similarly characterized by sparsity;
- $L_2$ - regularization - specifies the value of the total energy of the weights, this type of regularization was developed by A.N. Tikhonov. Within the regularization of Tikhonov, strictly zero weights in the optimal solution are practically impossible.

Thus, the search for a solution refers to the field of machine learning using a given dictionary, within which the search for an approximation of the input function, i.e., a seismic trace, is performed.

The most common method for solving spectral inversion is the Matching pursuit algorithm proposed by Mallat S. and Zhang Z [9]. The essence of the algorithm is reduced to an iterative search for the elements of the dictionary, minimizing the approximation error at each step. When using an orthogonal basis, this method represents an "orthogonal matching pursuit" [3,4]. This method of solving the spectral inversion problem relates to  $L_0$ -regularization and provides the most sparse solution.

### IV. METHODOLOGY

As noted earlier, the main purpose of the spectral inversion (SI) algorithms is to approximate the input seismic trace by a set of wavelets from a given library. The technique proposed by the authors consists in using the results of approximation for the formation of a path of the dominant frequency value over a priori defined wavelets. At the output of the algorithm, each trace of the wave field corresponds to a trace of the dominant frequency value, analogous to the time duration and the discretization.

The described technique raises a number of questions for its further application in practice. These questions can be divided into the following types:

- Formation of the wavelet dictionary;
  - Sensitivity of the algorithm;
  - Stability of the algorithm in the presence of noise.
- The authors have selected the following methodological framework, which allows to study the main questions of the proposed methodology (Figure 1):

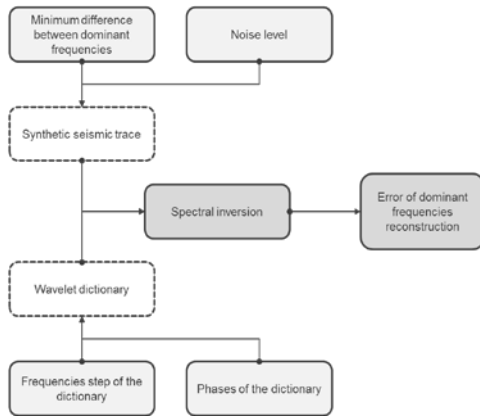


Fig. 1. Research framework.

The research is based on the acquisition of a series of model seismic traces with a different contrast of the dominant frequency and a signal-to-noise ratio. For their transformation into the curves of the dominant frequency, the OMP technique was used. At the same time, several practical questions of forming a wavelet dictionary for the realization of decomposition are considered: a sufficient step in the frequencies, as well as the influence of the phase composition. The latter factor reflects the influence of the seismic signal mismatch and the wavelets of the given library, which for real data may prove to be critically significant.

Thus, as the factors under consideration, the authors have selected the following:

1. The minimum difference between the dominant frequencies. Variants of close dominant frequencies with a difference from 1 to 5 Hz are considered;
2. Signal to noise ratio. The noise fraction is considered from 0 to 25% in 5% increments;
3. Step of the dictionary by frequencies. Dictionaries with a step from 1 to 5 Hz are considered;
4. Step through the phases for the dictionary. The range of phases from 0 to 180 degrees with a step of 5 degrees is considered.

As a criterion for assessing the quality of the solution, a comparison is made between the reconstructed and the model curve of the dominant frequency. Comparison at this stage of the study was carried out at a qualitative level.

## V. RESULTS

To perform the research, the authors consider a simple thick-layer model containing seven reflecting boundaries. The choice of such a model is due primarily to the elimination of interference at this stage of the study. The computation of the seismic model trace is accomplished by convolving reflection coefficients with a Ricker wavelet of a certain dominant frequency. For the background value of the dominant frequency, 30 Hz is assumed, for two signals in the middle part of the model the value of the dominant frequency changed in steps of 1 Hz upward. In addition, for each contrast model, a different content of random additive interference is considered from 0 to 50% in steps of 10%. Thus, the total number of models was 25 traces.

In addition to changing the parameters of the model, various approaches to the formation of the wavelet library are considered in the framework of the study. To form the library, Ricker's multi-frequency wavelets were used in the range of 20-40 Hz. In this case, a different step in frequencies from 1 to 5 Hz, as well as various phase characteristics of wavelets in the range of 0-180 degrees in steps of 5 degrees, is considered to assess the influence of inconsistencies in the wavelets of the seismic trace and the library. At the same time, within the framework of one iteration of calculations, the phase characteristics of wavelets within the library were constant. Thus, each model was investigated using 180 libraries.

The total number of digital experiments, taking into account the variability of models and decomposition libraries, was 900, and the main practical conclusions on the results obtained are given in the article.

The most predictable result is observed for the study of the minimal step of the library - as shown in Figure 2 when the pitch of wavelets is increased by more than 1.5 times, the possibility of determining the contrast of the dominant frequency is lost. This factor is important in optimizing the wavelet library, since the amount of time it takes to calculate the approximation directly depends on the number of elements it contains.

Consideration of the issue of minimum contrast is advisable to perform in conjunction with the resistance to noise. It is related to the fact that, according to the simulation results, it is established that in the absence of noise and at an agreed step in the library, the sensitivity on the model under consideration is preserved for arbitrarily small contrasts, which is not feasible in real data.

A qualitative analysis of the results of the applied method makes it possible to draw the following practical conclusions. For a significant contrast of the dominant frequency (above 5 Hz), a stable solution is observed even at a noise level of more than 50%. The minimum sensitivity threshold can be adopted at 1 Hz, since the appearance of even an insignificant share of random interference (up to 10%) leads to the impossibility of restoring the values of the dominant frequency along the path. In general, the simulation results show a logical result - with increasing noise and decreasing contrast at the dominant frequency, the sensitivity of the method decreases.

Table 1 shows the results of a qualitative assessment of the reconstruction of the dominant frequency model by the proposed algorithm. The green color of the cell corresponds to the qualitative recovery of the model, in which the contrast layer and the background values are determined correctly, the yellow color of the cell corresponds to the partial reconstruction of the model, the red corresponds to the incorrect reconstruction.

Table 1 Qualitative evaluation of the reconstruction of the dominant frequency model for different contrast and noise level.

		Noise level, % from maximum amplitude of the signal				
		0	12,5	25,0	37,5	50
Contrast of the dominant frequency	1 Hz					
	2 Hz					
	3 Hz					
	4 Hz					
	5 Hz					

In addition to those previously considered, there is an important issue is the consistency of the wavelets of the library and the seismic trace. To study this factor, a phase rotation of the library's wavelets was carried out to a constant value from 0 to 180 degrees in steps of 5 degrees, while the investigated model was composed of zero phase wavelets. Thus, the effect of the discrepancy between the shape of the seismic pulse and the wavelet library was modeled. The maximum error of the reconstruction was observed during the phase rotation of the wavelets by 90 degrees, in this case the wavelet of the library with model seismic signals is the greatest difference (Figure 4).

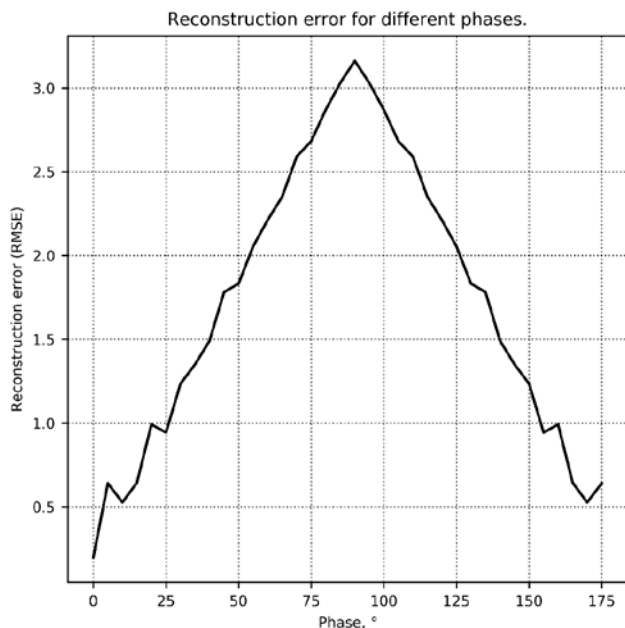


Fig. 2 Error of reconstructing the model trace by the Ricker wavelet library with a different phase spectrum.

In the results obtained, it is established that in the absence of a noise component, the presence of contrast is uniquely determined even for weak contrasts (less than 2 Hz). In the case of a 2 Hz contrast and a difference in the phase component of 90 degrees, there is an error in determining the absolute value and localization of the boundary (Figure 5).

The results of the investigation make it possible to obtain an important practical consequence that the correct implementation of the algorithm does not require an exact knowledge of the wavelet form. This effect makes it possible to significantly simplify the process of forming a wavelet library by using analytic pulses instead of wavelets extracted from the seismic trace.

## VI. CONCLUSION

As a result of the study, a new method for predicting the dominant frequency over time on the seismic trace was developed and tested using the method of spectral inversion. The presented method was tested on model seismic traces, showing the applicability of the algorithm for solving the task. As a result of the research, the following properties of the proposed method are established:

- The sensitivity is about 1 Hz. A shallower contrast is not determined if there is an insignificant noise component (up to 10%). Such sensitivity is sufficient for solving practical problems [10];
- The optimal step of the library should be coordinated with the necessary sensitivity, not exceeding it by more than 1.5 times;
- For significant contrasts of the dominant frequency (more than 5 Hz), the method provides a stable solution even in conditions of a significant noise content (above 50% of noise);
- The contrast of the dominant frequency is determined regardless of the consistency of the wavelets of the trace and the library. Thus, the implementation of the algorithm does not require an exact knowledge of the form of the wavelet. There is an error in determining the absolute value of the frequency, while maintaining the character of the relative change in frequencies along the path.

The proposed method can be used to assess the absorbing properties of the medium in order to predict the geological structure. Among the influencing factors, one can distinguish the quality factor of the medium, the fracturing, and also the effects associated with wedging and interference.

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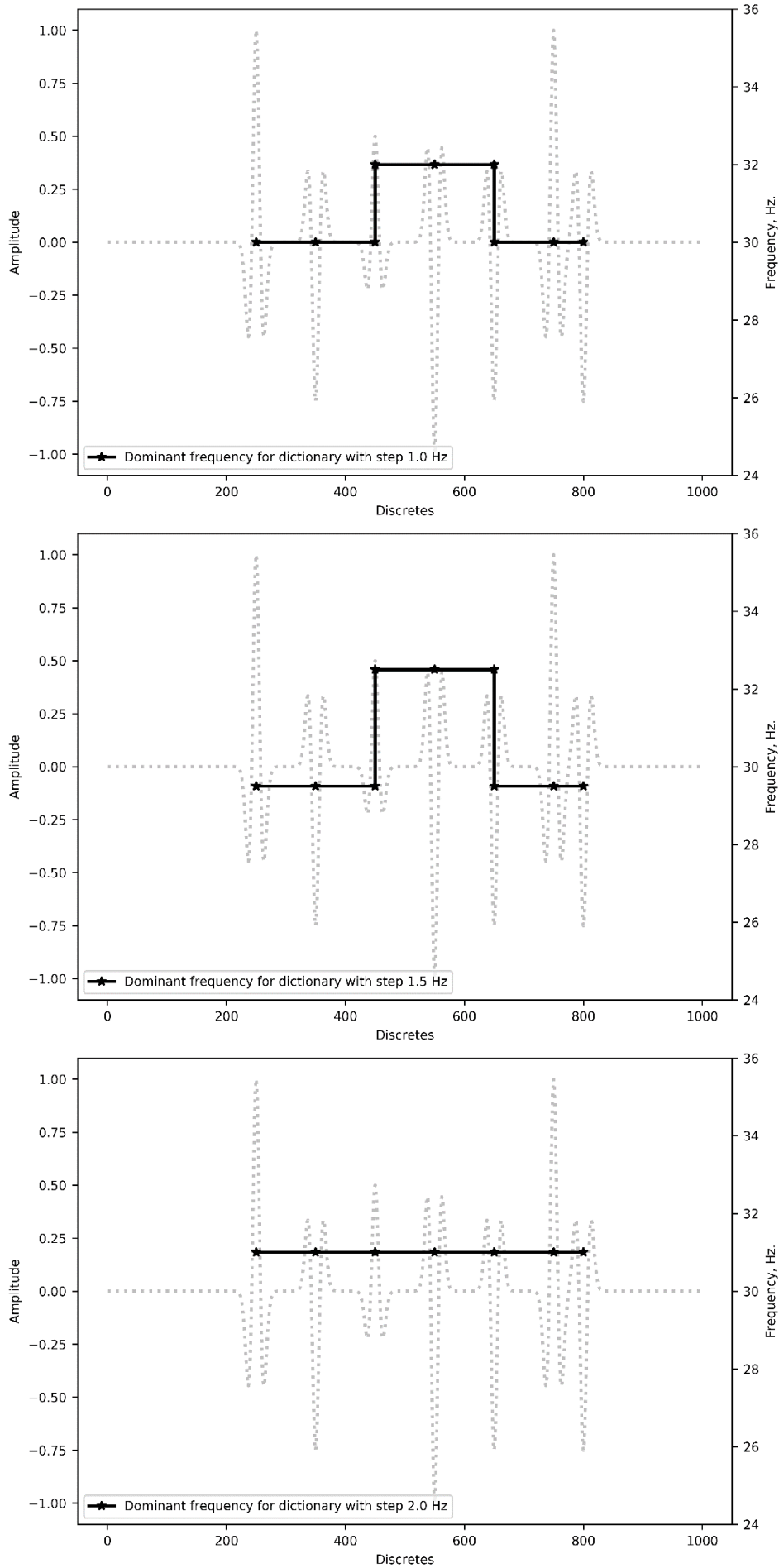


Fig. 3 Results of the reconstruction of the dominant frequency curve for the contrast of 2 Hz, using a library with a wavelet pitch of 1 Hz (top), 1.5 Hz (center), and 2 Hz (bottom).

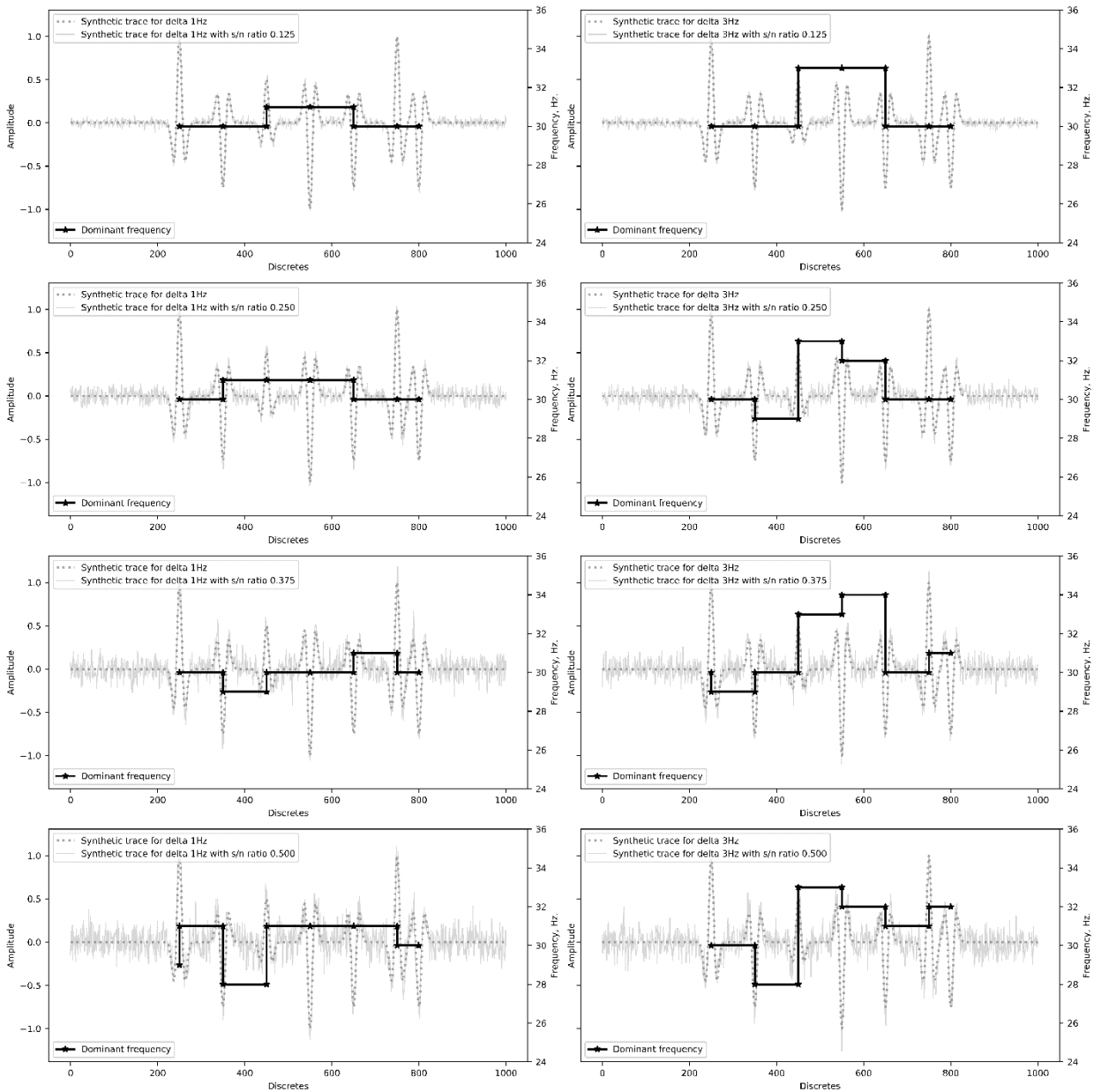


Fig. 4 The results of the reconstruction of the dominant frequency curve for the contrast of 2 Hz (left) and 4 Hz (right) for a different fraction of noise 12.5, 25, 37.5 and 50% (from top to bottom).

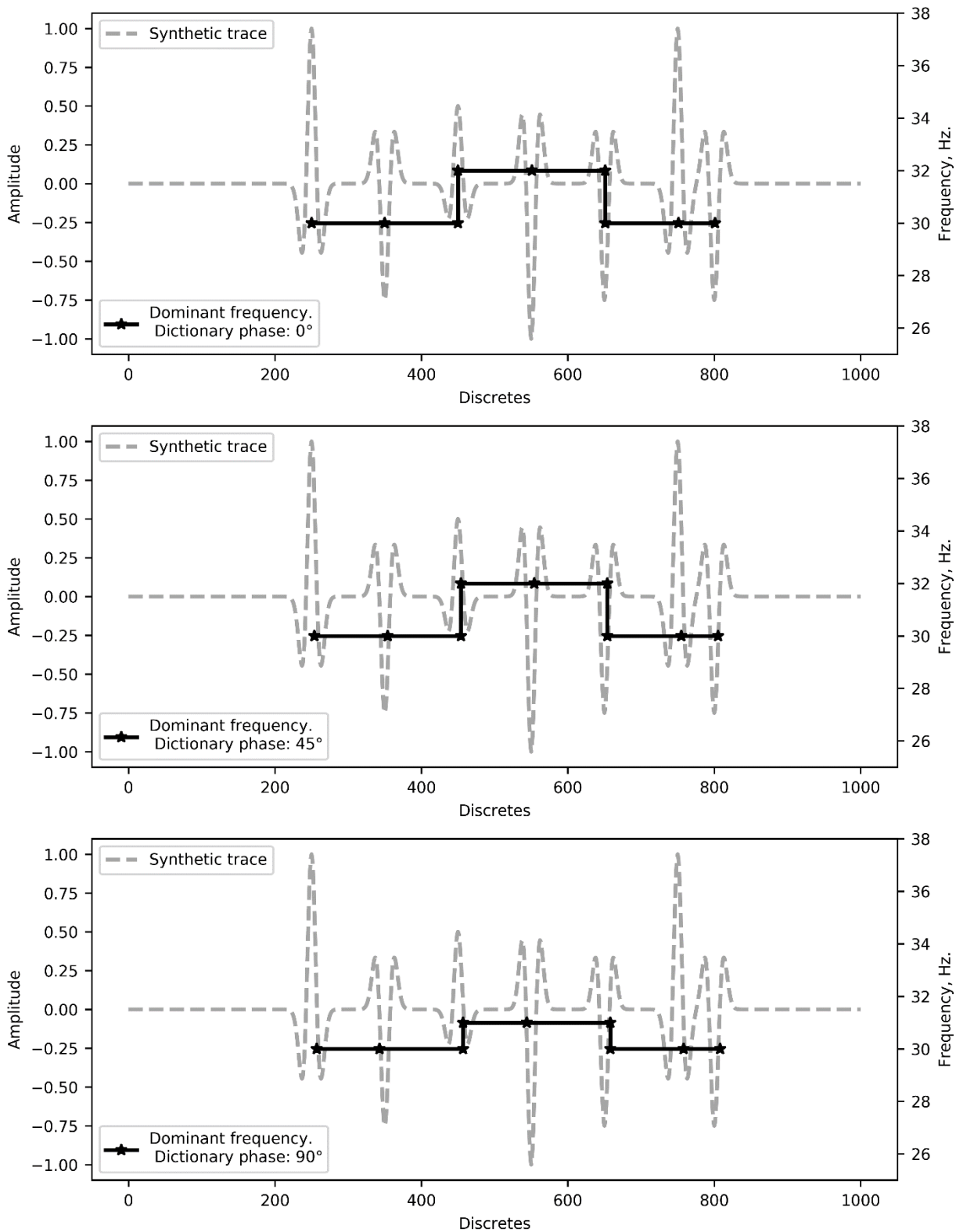


Fig. 5 Results of the reconstruction of the dominant frequency curve for 2 Hz contrast using Ricker wavelets with phase spectrum 0 (top), 45 (center) and 90 (bottom) degrees.

# Спектральная инверсия для оценки изменения доминантной частоты волнового поля

Федор Краснов, Александр Буторин

*Аннотация* — В статье предложен метод оценки доминантной частоты волнового поля вдоль сейсмической трассы, разработанный на основе метода спектральной инверсии. Основой предлагаемого алгоритма является оценка доминантной частоты по результатам аппроксимации сейсмической трассы вейвлетами из заданной библиотеки. Описанное исследование было направлено на определение основных требований к формированию вейвлет-библиотеки, а также на оценку чувствительности и устойчивости предложенного алгоритма в условиях различных значений контрастности на доминирующей частоте и уровня аддитивной случайной интерференции.

Практическая ценность предлагаемого подхода заключается в получении дополнительных характеристик наблюдаемого волнового поля для геологической интерпретации сейсмических данных. К числу ключевых факторов, влияющих на изменение доминантной частоты, относятся: поглощающие характеристики среды в зависимости от структуры исследуемого объекта, интерференционные эффекты, приводящие к изменению частоты суммарного импульса, и влияние трещиноватости.

В результате проведенных исследований даны практические рекомендации по реализации предложенного алгоритма, а также чувствительности и помехоустойчивости метода.

*Ключевые слова*— модель сигнала, свертка, ОМР, обучение со словарем.