Mathematical Modeling of the Fiber-Optic Converter on the Magneto-Optical Faraday Effect

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***Abstract*—This work we are engaged in research in the field of fiber –optic sensors, fiber - optic converters, magneto –optics and mathematical modeling. In recent years, a new type of fiber-optic measuring devices and systems on the magneto-optical Faraday effect was shown which was an important class of scientific equipment due to special advantages: very low inertia, passivity of the sensitive elements (no power supply); immunity; miniaturization, remote measurement with high sensitivity and performance. They may be in demand in the different fields of modern instrumentation.**

***Keywords*—Smart transmitter, numerical aperture, Faraday effect, number of turns of the optical fiber, mathematical model, intensity of radiation, voltage of the external magnetic field.**

I. Introduction

The Faraday effect describes that in the presence of the magnetic field H a circular birefringence arose in an optical fiber will rotate the plane of polarization of linearly polarized light at an angle of φF (Figure 1):

$$\varphi F = VH \text{ external}, \quad (1)$$

Where V – specific Faraday rotation (Verdict constant); L – length of the optical fiber, L=2πr N (r – radius of the fiber loop, N – number of turns of the fiber), actually found a wide application at the beginning of the XXI century [1]. In modern literature similar technologies gained a wide popularity in 2005 when N xt Phase Corporation (Canada) introduced one of the first working variants of the optical current converters [2]. Despite the relatively high cost of this technology, it is undoubtedly one of the key directions for the development of modern fiber optic devices and systems.

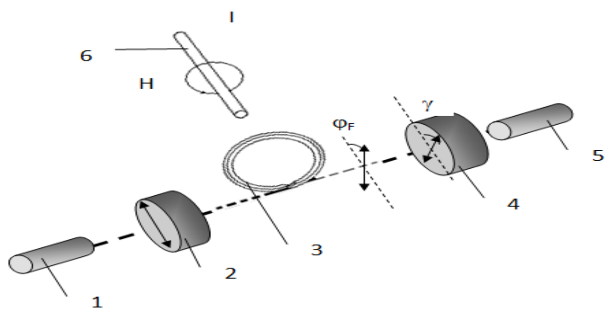

Figure 1 – Magneto-optical Faraday effect in optical fiber

Consider the physical model of the fiber optic Device on the magneto-optical Faraday Effect (FODMFE). Its structural scheme (Figure 2) is a compound of the main elements of the converter and it reflects physical processes occurring in it during the passage of light emission along the optical path, and the intensity of the external magnetic field created by a current -carrying conductor in the converter sensing element in which the interaction between these signals occurs.

FODMFE works as follows. When the electric current I flows through a conductor a controlled magnetic field with intensity H is created. This field affects the Faraday element (twisted optical fiber) FE. The coherent monochromatic radiation J1 generated by the laser diode LD in the polarizer P polarizes into linearly polarized light wave J2. In the Faraday element under the influence of an external magnetic field the rotation of the plane of polarization of the light takes place spreading along the magnetic field. Luminous flux J3 from the Faraday element output passes through the analyzer A and enters the photodiode PD followed by the measuring unit MU (Figure 2) which consists of the amplifier A, having further converting the signal via analog-digital converter ADC, and the liquid crystal display LCD obtains the value of the controlled magnetic field. Polarizer P and analyzer A are in the crossed position. Thus, it is possible to fix the slight change of intensity of the luminous flux entering the photodiode [4].

The most important element in this scheme is the sensitive FE in which under the influence of the magnetic field on some materials in them an induced optical activity arises. This effect is achieved by interaction of the magnetic fields of light and electronic orbitals [5].

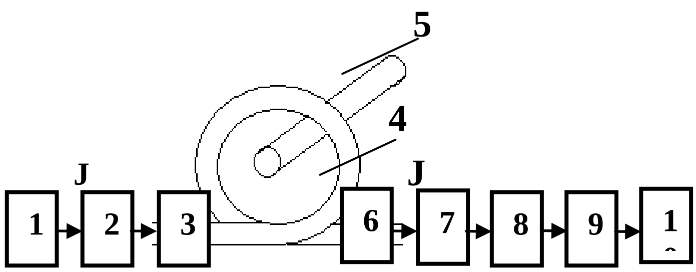

Figure 2 - FODMFE block diagram

1 - Laser diode; 2 – connecting optical fiber; 3 - polarizer; 4 - sensing element (turn of optical fiber); 5 - current-carrying conductor generates a magnetic field; 6 - analyzer; 7 - photodiode (PD); 8 - operational amplifier (OA); 9 - analog - digital converter (ADC); 10 - liquid crystal display (LCD).

Fiber optic devices and systems based on the magneto-optical Faraday Effect can be built on the principle of the intelligent system. For this purpose one must add a neural sensor and a control device to the block diagram 2. The neural sensor processes the data obtained from the analog -

digital converter after which the data enter the control device which regulates the current, all the data are represented on the liquid crystal display.

The device operates as follows (Figure 3). When the electric current I flows through the conductor 5 a controllable magnetic field of intensity H is created. This field affects the Faraday element (twisted optical fiber) 4. Coherent monochromatic radiation J1 generated by the laser diode 1 passes through the connecting fiber 2, then it comes into the polarizer 3 where it is polarized in the linearly polarized light wave J2. In the Faraday element 4 under the influence of an external magnetic field is the rotation of the polarization plane of light propagating along the magnetic field.

Luminous flux J3 from the Faraday element output passes through the analyzer 6 and falls on the photodiode 7, then on the measuring block 8 which consists of an amplifier, hereinafter, the signal is converted by the analog-digital converter 9. The obtained values of the controlled magnetic field come into the neural sensor 10 where there is an interaction of the real-time data analyzer 11, environment of the intellectual measuring system 12, smart transmitter 13, Human Machine Interface 14, control (reactive part) 15. After data processing the control device 16 operates to regulate the current flowing through the conductor 5. On the LCD 17, we obtain a value of the controlled magnetic field [2].

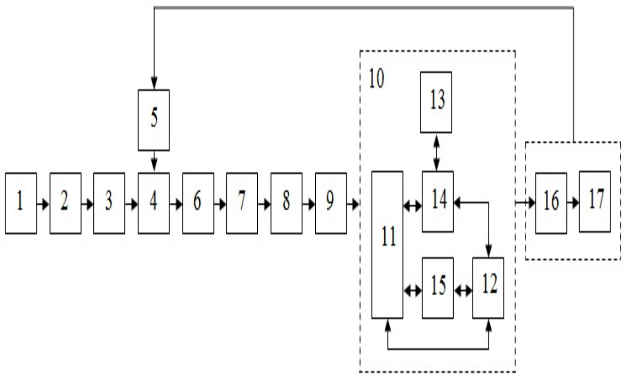

Figure 3. Block diagram of the smart transmitter of the magnetic field and electric current on the Faraday Effect

Modern quartz optical fibers are a diamagnetic material which has a unique opportunity to use it as a sensor device. In circumstances when there is no magnetic field, the intensity of light radiation coming out of the analyzer is:

$$J_4 = J_2 \cos^2 \gamma, \qquad (2)$$

J2 -light intensity for the polarizer; γ - interlunar angle of the polarizer and analyzer polarization (angle of crossing), which is installed by the device setting from 00to 900previously.

The plane of polarization of linearly polarized light in the FE under the influence of an external magnetic field is further rotated by an angle (formula 1). Dependence of the Verdict constant of the wavelength can be determined as follows.

Consider the influence of an electromagnetic wave on the studying electron. While studying FODMFE it is necessary to consider in the optical sensor system the action of the external field which is many times larger than the intensity of the magnetic field of the electromagnetic wave.

Therefore, it is necessary to use the Lorentz force to produce the dependence of the refractive index of any substance from the frequency of incident light in order to find a vector of polarization P of the substance produced by the light wave field E.

Calculate the electrostatic induction vector $D = E + 4\pi P$ and define $\varepsilon = \frac{D}{E}$.

Use the basic relation of the electromagnetic theory of light $n = \sqrt{\varepsilon}$, obtain the Lorentz force, recorded as

$$F_л = q\left[E + \frac{1}{c}\left[v \cdot H_{external}\right]\right]. \qquad (3)$$

In case of limited range of optical spectrum (infrared, visible and ultraviolet radiation), even at a very low density of matter there is an enormous amount of emitting atoms, and one can assume that the polarization of the substance in the light wave field is given by

$$P = Nqr. \qquad (4)$$

N - Number of molecules per unit of volume, the product of the electric charge q at the radius vector r - electric dipole moment.

Consider that at the "input" into the substance there are two waves, circularly polarized:

$$E_x = E_0 \cos \omega t \quad E_y = \pm E_0 \sin \omega t \qquad (5)$$

Influence of the field direction on the polarized wave propagation can be considered with the right and left rotation

$$E_x + iE_y = E_0 e^{(i\omega t)} \quad \text{(right rotation)},$$

$$E_x + iE_y = E_0 e^{(-i\omega t)} \quad \text{(left rotation)},$$

Using the formula of the medium polarization (4) and the expression

$$n^2 = \varepsilon = 1 + \frac{4\pi P}{E}$$

Find

$$n_{\pm}^2 = 1 + \frac{4\pi N \cdot q^2 / m}{(\omega_0^2 - \omega^2) \pm q \cdot \omega \cdot H_{external} / (mc)} \qquad (6)$$

For the wave propagation in the test medium, there are two values of the refractive index n left and n right, as in an optically active substance the wave propagation velocity with the right rotation is different from the wave propagation velocity with the left rotation [6]. The explicit expression (6) can be written as: n left and n right

$$n_{left} - n_{right} = \frac{4\pi N q^3}{n m^2 c^2} \frac{H_{external}\omega}{(\omega_0^2 - \omega^2)^2}. \qquad (7)$$

Using the formula of the polarization plane rotation connecting the difference of the refractive indices with an angle of rotation φ of a polarization plane for a wave which has passed the path l in the test medium, we find

$$\varphi_F = \frac{\omega}{2c}(n_{left} - n_{right})l = \frac{2\pi N q^3}{n m^2 c^2} \frac{\omega^2 l H_{external}}{(\omega_0^2 - \omega^2)^2}, \qquad (8)$$

Where

$$\frac{2\pi N q^3}{n m^2 c^2} \frac{\omega^2}{(\omega_0^2 - \omega^2)^2} = V.$$

At $\omega << \omega_0$ (electronic absorption bands lie in an ultraviolet spectral range) it is possible to neglect ω^2 in a denominator of the last expression and we receive Verde constant dependence on frequency $\rho \cong \omega^2 \cong 1/\lambda^2$. Finally, Verde constant can be found by the formula

$$V = \frac{q}{2mc^2}\omega\frac{\partial n}{\partial \omega} \qquad (9)$$

Using the formula (1) let us write the expression (2) in the form:

$$J = J_4 = J_2 \cos^2\left[\gamma + \left(\frac{q}{2mc^2}\omega\frac{\partial n}{\partial \omega}\right) \cdot H_{external} \cdot (2\pi r N)\right] \qquad (10)$$

The formula (10) describes amplitude modulation of the linearly polarized light by the controlled magnetic field created by the electric current.

The intensity of light falling on a polarizer J2 (see figure 2) is defined by pointing vector.

The intensity which has left the analyzer,

$$J = \frac{1}{2} J_0 \cdot e^{(-\alpha l)} \cdot \cos^2\left[\gamma + \left(\frac{q}{2mc^2}\omega\frac{\partial n}{\partial \omega}\right) \cdot H_{external} \cdot (2\pi r N)\right] \qquad (11)$$

Where α=2ωk/c= (4π/λ) k - extinction coefficient.

Expression (11) is given without taking into account losses of light intensity in the reflection from surfaces of optical fibers, as well as in the assumption $\alpha = const$ at a distance l_1 from the light source.

To build FODMFE intended for the measurement of an alternating current, it is more advisable to run the system with the account of $\gamma = 45^0$.

The loss of light on the reflection from fiber surfaces when passing through the optical system FODMFE and absorption in the polarizers are inevitable in the real system, therefore, to account for these light losses it is needed to enter the ratio g in the formula (11)

$$g = 2\chi_{reflection} + 2q_{absorption}, \qquad (12)$$

Where χ reflection - the reflection coefficient of a polarizer; q absorption - the absorption coefficient of a polarizer.

Light movement in an optical fiber will be influenced by the core or internal surface of the light guide which has a high rate of refraction and is defined by a numerical aperture

$$NA = n_1\sqrt{2\Delta} \qquad (13)$$

It is one of the most important parameters.

The optical beam extends in an optical fiber, experiencing multiple total reflection from the core and envelope transition. However, this does not mean that the angle can be unspecified practically. On the contrary, the wave angle of a beam is the singular, characteristic one.

Consequently, the formula (11) takes the form:

$$J = \frac{1}{2} J_0 \cdot e^{(-\alpha l)} \cdot g \cdot NA \cdot \left(\cos^2\left[\gamma + \left(\frac{q}{2mc^2}\omega\frac{\partial n}{\partial \omega}\right) \cdot H_{external} \cdot (2\pi r N)\right]\right) \qquad (14)$$

And using the formula for the instantaneous value H in the far zone, rewrite the formula (14) as follows:

$$J = \frac{1}{2} J_0 \cdot e^{(-\alpha l)} \cdot g \cdot NA \cdot \left(\cos^2\left[\gamma + \left(\frac{q}{2mc^2}\omega\frac{\partial n}{\partial \omega}\right) \cdot \left(\frac{I_m dl \sin\Theta}{2R}\right) \cdot (2\pi r N)\right]\right) \qquad (15)$$

The formula (15) is a mathematical model of FODMFE - the intensity dependence of the optical radiation at the output of the sensing element on the external magnetic field of an electrical conductor attenuating properties of the material and type.

Based on the formula (15), consider calculated depending on the intensity of the radiation at the output of the sensing element FODMFE, on parameters of the optical system, shown in Figure 4, 5,6.

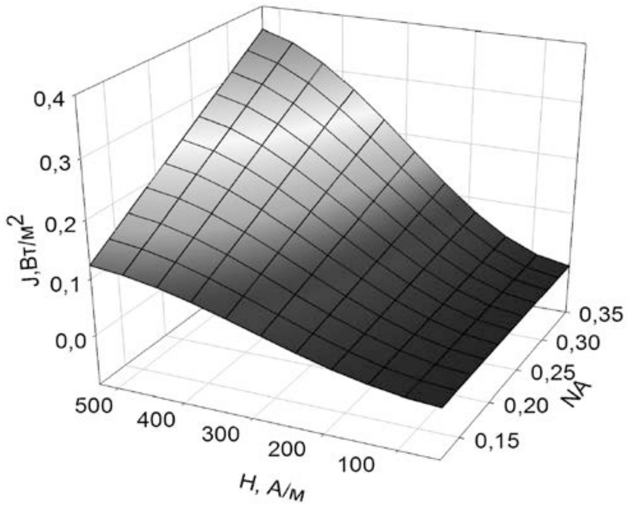

Figure 4. Dependence of the intensity of radiation J at the output of the sensing element FODMFE on the voltage of the external magnetic field H with various parameters NA (0,12ч0,35) and the number of turns of the optical fiber N: a) N=500

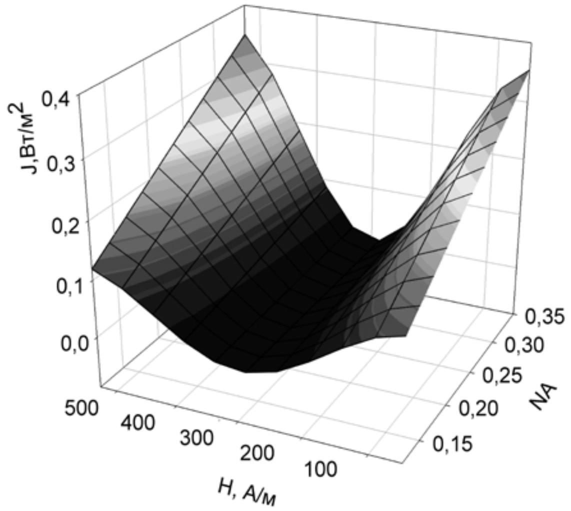

Figure 5. Dependence of the intensity of radiation J at the output of the sensing element FODMFE on the voltage of the external magnetic field H with various parameters NA (0,12ч0,35) and the number of turns of the optical fiber N: b) N=1000

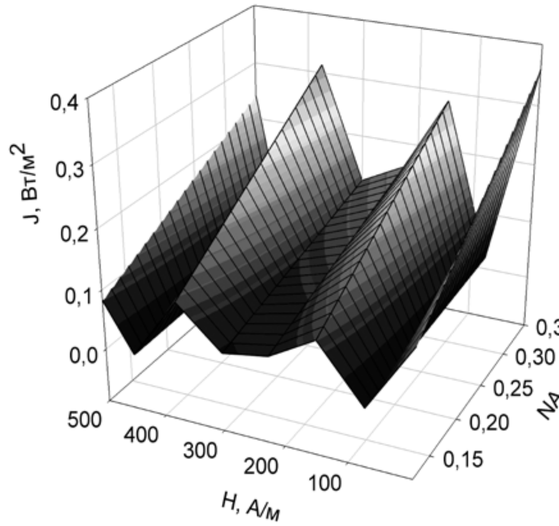

Figure 6. Dependence of the intensity of radiation J at the output of the sensing element FODMFE on the voltage of the external magnetic field H with various parameters NA (0,12ч0,35) and the number of turns of the optical fiber N: c) N=1500

CONCLUSIONS

The model showed that the greatest impact on the value of the radiation intensity by fiber parameters - numerical aperture (NA) and the number of fiber turns (N). Therefore, it is necessary to simulate the process of changing the intensity of the optical radiation propagating through the optical system of the converter.

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