Classification of natural numbers based on arithmetic progressions with a difference 6.

G. G. Ryabov, V. A. Serov

***Abstract*—The article considers a classification of natural numbers based on the submission of the set of all natural numbers as union of six infinite arithmetic progressions. The classes themselves (bijective to the progressions) are considered as members of two finite semigroups with regard to the operations of addition and multiplication. The binary relations between classes and examples of natural numbers properties at such classification are given.**

***Keywords*—Arithmetic progression, bijection, binary relations, classification, Goldbach prime, set of natural numbers, semigroup.**

I. Introduction

The representation of the set of all natural numbers greater than or equal to 4 $N(\geq 4)$ as the combination of six infinite arithmetic progressions denoted as $S_4 = \{4+6m\}$, $S_5 = \{5+6m\}$, $S_6 = \{6+6m\}$, $S_7 = \{7+6m\}$, $S_8 = \{8+6m\}$, $S_9 = \{9+6m\}$, $m \in N_0$ was considered in [1]-[3].

In this representation, every natural n_i has a unique expression in the form of pairs of numbers (r_i, m_i), $i \in N$, where r_i is the index of progression, coinciding with the initial member of progression, and m_i is the ordinal number of the number in this progression. It was also proved that all prime numbers P greater than 4, $P(>4)$ are contained only in two progressions of these six, namely $S_5 = \{5+6m\}$ and $S_7 = \{7+6m\}$.

Let us agree within the article not to introduce special notations for the proposed classes, which are bijective to six progressions, because substantial part lies in the number-the index of progression, which will be used in the expressions of binary relations between classes in the form of 6×6 tables.

In this article, when ensuring [1] approach, the following six progressions will be considered: $S_3 = \{3+6m\}$; $S_4 = \{4+6m\}$; $S_5 = \{5+6m\}$; $S_6 = \{6+6m\}$; $S_7 = \{7+6m\}$; $S_8 = \{8+6m\}$;

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G.G. Ryabov is with the Research Computing Center, M.V. Lomonosov Moscow State University, Moscow, Russia (e-mail: gen-ryabov@ yandex.ru).

V.A. Serov is with the Research Computing Center, M.V. Lomonosov Moscow State University, Moscow, Russia.

The initial fragment of this six progressions is given below (accordingly, and the offered classes natural) Fig.1. Primes in them are marked more in bold, and primes-twins also by underlining.

Properties of progressions go into properties of classes that act as elements of the semigroups under addition and multiplication. So, it should be noted the behavior of primes degrees within this six progressions included in the canonical multiplicative representation of numbers, equal to the index of progressions. So, for S_3—3, S_4—2, S_5—5, S_6—6, S_7—7, S_8—2 , respectively. It is easy to verify by elementary calculations the following properties inherent to each of these progressions and missing all the rest.

1. S_3 contains all (even and odd) degrees of 3. $\{3, 9, 27, 81, 243, 729, 2187, \ldots\} \in S_3$.
2. S_4 contains only even powers of 2. $\{4, 16, 64, 256, 1024, 4096, \ldots\} \in S_4$.
3. S_5 contains only odd powers of 5. $\{5, 125, 3125, 78125, \ldots\} \in S_5$ (1)
4. S_6 contains all (even and odd) degrees of 6. $\{6, 36, 216, 1296, \ldots\} \in S_6$.
5. S_7 contains all (even and odd) degrees of 7. $\{7, 49, 343, 2401, \ldots\} \in S_7$.
6. S_8 contains only odd powers of 2. $\{8, 32, 512, 2048, \ldots\} \in S_8$.

This property can be considered as an invariant of progression and, hence, an invariant of bijective class of the natural. On the other hand, it can be used as a marking of an infinite arithmetic progression by elements of an exponential nature (as automorphic built-in logarithmic scale).

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
$S_3=$	3	9	15	21	27	33	39	45	51	57	63	69	75	81	87	93	99	105	111	117	123	129	135	141	147
$S_4=$	4	10	16	22	28	34	40	46	52	58	64	70	76	82	88	94	100	106	112	118	124	130	136	142	148
$S_5=$	**5**	**11**	**17**	**23**	**29**	35	**41**	**47**	**53**	**59**	65	**71**	77	**83**	**89**	95	**101**	**107**	**113**	119	125	**131**	**137**	143	**149**
$S_6=$	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
$S_7=$	**7**	**13**	**19**	**25**	**31**	**37**	**43**	49	55	**61**	**67**	**73**	**79**	85	91	**97**	**103**	**109**	115	121	**127**	133	**139**	145	**151**
$S_8=$	8	14	20	26	32	38	44	50	56	62	68	74	80	86	92	98	104	110	116	122	128	134	140	146	152

Fig.1. The initial fragment of six progressions $S_3 - S_8$.

II. BINARY RELATIONS BETWEEN CLASSES AS ELEMENTS OF SEMIGROUPS

Classes of natural, bijective to these progressions, are in binary relations to the operations of addition and multiplication, which are shown in a tabular form in Fig.2. It means that for any pair of natural such that $n_1 \in S_{r_1}$ and $n_2 \in S_{r_2}$, the sum $(n_1 + n_2) \in S_{r_3}$, where r_3 is the index at the intersection of the r_1-th column and the r_2-th row (Fig.2a) (or vice versa—the row and the column, since both addition and multiplication are commutative), and the tables are symmetric. Similarly, for multiplication (Fig.2b).

In the semigroup under addition the role of the "0" makes the class S_6, and in the semigroup under multiplication the role of "1" performs the class S_7.

+	S3	S4	S5	S6	S7	S8
S3	6	7	8	3	4	5
S4	7	8	3	4	5	6
S5	8	3	4	5	6	7
S6	3	4	5	6	7	8
S7	4	5	6	7	8	3
S8	5	6	7	8	3	4

a)

X	S3	S4	S5	S6	S7	S8
S3	3	6	3	6	3	6
S4	6	4	8	6	4	8
S5	3	8	7	6	5	4
S6	6	6	6	6	6	6
S7	3	4	5	6	7	8
S8	6	8	4	6	8	4

b)

Fig.2. a). The table of binary relations between the classes $S_3 - S_8$ under addition. b). The table of binary relations between the classes $S_3 - S_8$ under multiplication. The cells corresponding to the class S_6 are darkened.

Given in the form of tables binary relations between classes can be used to calculate tables for higher arities. Here we will limit ourselves to the example tables of ternary relationships between classes. Such tables are no longer flat but three-dimensional $6 \times 6 \times 6$ (Fig.3). For a flat rendering will use the so-called "slice-type".

We are interested in the question: To which class belongs the operation result of "random" pairs or triples of natural. It was natural therefore to consider the table of binary relations as the set of possible outcomes at a probability of 1/6 of each class of the operand. Then the ratio of the number of indexes of each class to 36 is equal to the probability to get the result in this class of natural.

For binary relations under addition the probability of the result from each class is equal to 1/6, since $\#(r_i, T_b) = 6$. For binary relations under multiplication we have different picture, because:

$\#(3, T_b) = 5$; $\#(4, T_b) = \#(8, T_b) = 6$; $\#(5, T_b) = \#(7, T_b) = 2$; $\#(6, T_b) = 15$;

Hence the probability of the product of two randomly selected natural to be in the class $p_{\times}(r_i, b)$:

$p_{\times}(3, b) = 5/36$; $p_{\times}(4, b) = p_{\times}(8, b) = 1/6$;
$p_{\times}(5, b) = p_{\times}(7, b) = 1/18$; $p_{\times}(6, b) = 5/12$;

Similar calculations for ternary relations under multiplication give the following results:

$\#(3, T_t) = 19$; $\#(4, T_t) = \#(8, T_t) = 28$; $\#(5, T_t) = \#(7, T_t) = 4$; $\#(6, T_t) = 133$;

Probability to the product of three randomly selected natural to be in the class $p_{\times}(r_i, t)$:

$p_{\times}(3, t) = 19/216$; $p_{\times}(4, t) = 7/54$; $p_{\times}(5, t) = 1/54$;
$p_{\times}(6, t) = 133/216$; $p_{\times}(7, t) = 1/54$; $p_{\times}(8, t) = 7/54$;

At the construction of natural of some random class in the degree the response to a similar question about the probability of the class for the result is contained in the table on the main diagonal. In particular for the construction of the natural square (a binary relation under multiplication) the diagonal is a set $D_{2b} = \{S_3, S_4, S_7, S_6, S_7, S_4\}$. $S_5, S_8 \notin D_{2b}$, which corresponds to (1).

Thus $p_2(3, b) = 1/6$; $p_2(4, b) = 1/3$; $p_2(5, b) = 0$;
$p_2(6, b) = 1/6$; $p_2(7, b) = 1/3$; $p_2(8, b) = 0$;

Similarly: $D_{3t} = \{S_3, S_4, S_5, S_6, S_7, S_8\}$ and
$p_3(3, t) = p_3(4, t) = \ldots = p_3(8, t) = 1/6$;

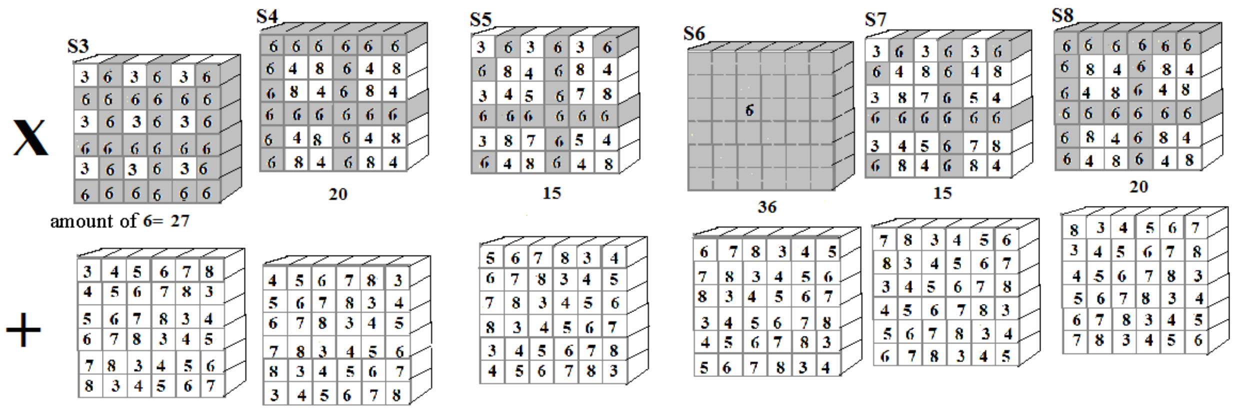

Fig. 3. The tables of ternary relationships between classes under multiplication (the top row) and under addition (the lower row). In the top row the cells corresponding to the class S_6 are darkened, to emphasize the property of idemponentiality.

III. New classification and the Goldbach primes

Prime pairs g_1 and g_2, the total of which is equal to a given even natural number, play a significant role in researches related to the binary Goldbach problem. These pairs are called the Goldbach primes.

In the proposed classification, as shown in [1], all primes more 4 belong to the classes S_5 and S_7. As well as compound of these classes they satisfy to the binary relations under addition (Fig.2). All even greater than or equal to 4 belong only to the classes S_4, S_6, S_8. It is easy to see (Fig. 4) that even from the class S_4 are Goldbach prime numbers conjecture only from S_5 ($G(S_4) \in S_5$), and the even-numbered from S_8 only from S_7 ($G(S_8) \in S_7$). Even from S_6 always have one Goldbach prime from S_5, and the other from S_7, including this applies to primes-twins.

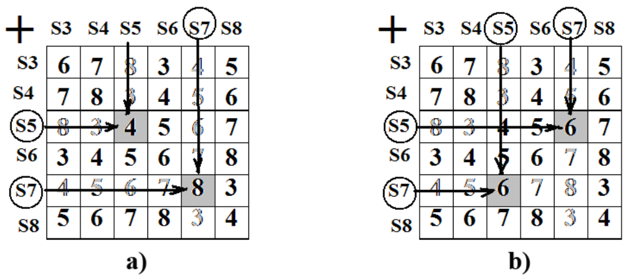

Fig 4. a). Dedicated binary relations under addition: $(S_5 + S_5) \in S_4$ and $(S_7 + S_7) \in S_8$; b). Selected binary relations $(S_5 + S_7) \in S_6$ and $(S_7 + S_5) \in S_6$;

IV. Conclusion

The General structure of the proposed classification is shown in Fig.5, in which:

$P(S_5)$—primes of the class S_5; $P(S_7)$—primes of the class S_7; $P(S_5 \cup S_7)$— twin primes;

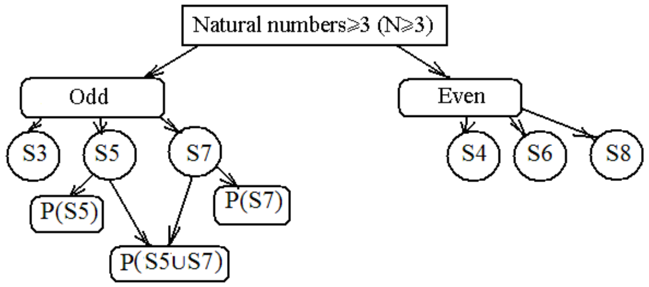

Fig 5. The General scheme of the proposed classification of natural numbers.

The above natural classification allows to estimate the number of joint properties of primes and compound when operating on very large natural numbers. And at the same time to raise the question about the architecture of superprocessor focused on work with very large integers.

References

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