Reconstruction of 2D structure of highly absorbing media

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***Abstract*— This paper presents a solution for the tomography of smoothly heterogeneous highly absorbing medium using the amplitude trajectories method. The law of the amplitude trajectory curvature in asymmetric medium is determined, which is similar to Snell's law in geometric optics. The solutions obtained apply for the cases of axisymmetric and asymmetric medium in the approximations of the amplitude trajectory straightness and its slight curvature due to the medium's heterogeneities. In the process of solving this problem, a new transform of Abel type has been discovered.**

***Keywords*— Abel transform, amplitude trajectories method, geometric optics, wave propagation.**

I. Introduction

Since the very moment tomography arose as a science, there has been a strong interest in using tomographic methods for reconstructing the inner structure of highly absorbing mediums. Firstly, the reason behind it is the existing demand for diagnostics of various tissue diseases. Microwave radiation usage is especially promising for diagnostics of early-stage pathologies because most diseases change the wetness of the affected tissues in the very onset, resulting in a change of their permittivity and conductivity, while their density stays the same.

Aside from technical issues of matching the emitter with the medium and receiving weak signals, there are problems with processing of gathered data and following reconstruction of the medium's inner structure. The major contribution into the field in the point observed is not made by the radiation scattered through heterogeneities since the absorption rate of the medium is high and the contrast of heterogeneities studied is low; actually, it is made by transmitted radiation, properties of which change after interacting with the medium. To reconstruct the medium's inner structure, it is necessary to analyze major effects of interaction between radiation and heterogeneities at each point of the medium and consider their integral impact on the recorded field's characteristics. Since radiation absorption in the medium is a dominant effect here, the integral attenuation of intensity is the main characteristic and phasic relations have a secondary role. Thus, the purpose of highly absorbing medium tomography is to define its internal structure by multangular projections of integral attenuation of the transmitted radiation intensity.

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There are great achievements in solving both direct and inverse sounding problems for non-absorbing media. In this case, processes in a medium with a fairly smooth characteristics change depending on coordinates can be efficiently described using ray representation [1]. The most commonly known method of solving inverse problems in geometric optics is solving the Abel equation that sums up the problems of through-transmission of planar layered and radially heterogeneous media.

If the absorption is low, it can be considered by slightly modifying the geometric optics equations using perturbation methods. In this case, the beam trajectory is determined by the dielectric constant's real part, and the imaginary part determines the integral attenuation of the radiation propagating along this curve [1]. Measuring the phase progression and attenuation of radiation caused by the medium provides us the ability to reconstruct its complex dielectric permittivity.

If the absorption is high, it is possible to use the complex geometric optics approximation which considers how absorption affects the beam trajectory. However, the problem of physical interpretation of complex rays still remains unsolved.

It is difficult to interpret the correlations discovered as a result of solving the direct problem, therefore, the precise solution of the inverse problem using this approach hasn't been found yet. Wave trajectories are considered to be straight lines in existing reconstruction methods. For highly heterogeneous media with a large linear absorption, the trajectory straightness can hardly be considered correct because even the very concept of a ray trajectory needs to be revised.

In [2], an amplitude trajectories method was proposed to describe the propagation of radiation in highly absorbing heterogeneous media. It is based on the assumption that the dominant effect determining the propagation conditions of the wave in the medium is not interference of wave disturbances, but attenuation in the medium. In this case, the wave that passes along the trajectory with minimal attenuation (amplitude trajectory) will have the greatest value of the field amplitude at the receiving point. A significant contribution into the resulting field in the observation point will be made only by those trajectories for which the attenuation differs from its extreme value by no more than e times. The higher the attenuation, the tighter the area of space is compressed near the extreme line, within which the corresponding virtual trajectories are concentrated. The zone that is essential for propagation can be identified as the amplitude trajectory of the wave.

The description of the wave attenuation along this curve is similar to the description of ray refraction in geometric optics. This allows us to reduce the problem of reconstructing the absorption coefficient profile by attenuating the transmitted wave intensity to the methods used for non-absorbing media and, particularly, to use the Abel transform and its generalizations for finding the solution. In [2], an estimate of the amplitude trajectory concept applicability was obtained: $n/n' > 1/m\pi$, where n', n are real and imaginary parts of the refractive index respectively, $m \approx 3 \div 5$ - number of first Fresnel zones taken into consideration.

II. Tomography of axisymmetric absorbing media

Consider the case of an absorbing medium with axial symmetry. The intensity of the field passing through such a medium can be asymptotically represented as a dependence given by $I = I_0 \exp\left(-2k_0 \int n(l)dl\right)$. Here I_0 denotes some slowly changing function, the dependence of which on the coordinates can be neglected, n denotes the imaginary part of the complex refractive index, k_0 denotes the wave number.

From the requirement of minimizing attenuation in a medium by the calculus of variations methods, one can obtain an equation for the amplitude trajectory in an axisymmetric medium,

$$n(r)r\sin\alpha(r) = n(r_0)r_0\sin\alpha_0 = p\,, \qquad (1)$$

equivalent to Snell's law for geometric optics.

It is reasonable to consider the imaginary part of the electric length $L = \int ndl$ as a measured quantity, which, taking into account (1), takes the form of

$$L(p) = 2\int_{r_{\min}(p)}^{r_0} \frac{(n(r)r)^2 dr}{r\sqrt{(n(r)r)^2 - p^2}}\,, \qquad (2)$$

where $r_{\min}(p)$ denotes the radius of the beam turning point, determined from the relation $n(r_{\min})r_{\min} = p$. As a parameter characterizing the distance between the enter and exit points of the amplitude trajectory, it is convenient to consider the angle (Fig. 1)

$$\psi(p) = 2p\int_{r_{\min}(p)}^{r_0} \frac{dr}{r\sqrt{(n(r)r)^2 - p^2}}\,. \qquad (3)$$

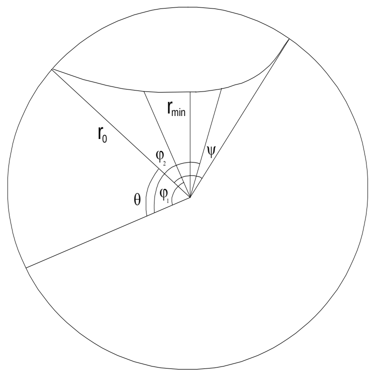

Fig. 1. The statement of the two-dimensional problem.

Thus, when the inverse problem is being solved, the dependence $L(\psi)$ becomes the measured quantity, and the radial profile of the refractive index imaginary part $n(r)$ acts as the desired function. In [2], it is shown that experimentally measured readings $L(\psi)$ in various points on the object surface can be associated with the impact parameter p, which is related to the beam entry angle α_0 according to the formula

$$p = dL/d\psi\,. \qquad (4)$$

By reducing the dependence (2) to the Abel transform, an equation for finding $n(r)$ [2] is obtained

$$\ln\frac{r}{r_0} = \frac{1}{\pi}\int_{n(r)r}^{n(r_0)r_0} d\psi \ln\left(\frac{p + \sqrt{p^2 - (n(r)r)^2}}{n(r)r}\right). \qquad (5)$$

Using this equation, the value r can be found from the given value of the parameter $n(r)r$, and this, ultimately, is equivalent to restoring the desired dependence $n(r)$. It should be noted that the value of the refractive index imaginary part on the surface $n(r_0)$ is considered to be known.

III. Attenuation of intensity in an asymmetric absorbing medium

Let us generalize the results in the case of a medium whose absorption coefficient depends on two coordinates. Let us choose the distance r from some fixed volume point to the current medium point and the angle φ between a selected direction and the direction to the point observed as these coordinates (Fig. 1).

To restore the two-dimensional dependence $n(r,\varphi)$, multi-angular measurements of the attenuation are necessary. Such measurements can be carried out by changing the position of the radiation source and conducting the reception at all points on the surface of the investigated object at each position of the source. The result of such measurements will be a two-dimensional dependence $L(\psi,\theta)$, where ψ denotes the angular distance between the source and the receiver, θ denotes the angle between the direction to the source point and the direction of the angle φ origin.

From geometrical considerations, we find the relations of the quantities L and ψ to the initial angle of incidence of the beam α_0:

$$L = \int_{r_{\min}}^{r_0} \left(\frac{n(r,\varphi_1)}{\cos\alpha(r,\varphi_1)} + \frac{n(r,\varphi_2)}{\cos\alpha(r,\varphi_2)} \right) dr, \tag{6}$$

$$\psi = \int_{r_{\min}}^{r_0} \frac{\operatorname{tg}\alpha(r,\varphi_1) + \operatorname{tg}\alpha(r,\varphi_2)}{r} dr .$$

Here φ_1, φ_2 denote the angular coordinates of space points on different parts of the trajectory, corresponding to the same value of r: $\varphi_1 \le \varphi_{\min}$, $\varphi_2 \ge \varphi_{\min}$, where $\varphi_{\min}$ denotes the angular coordinate of the turning point $(r_{\min}, \varphi_{\min})$ for a particular angle. Values φ_1 and φ_2 are also determined by the path of the beam:

$$\varphi_1 = \theta + \int_r^{r_0} \frac{\operatorname{tg}\alpha(r,\varphi_1)}{r} dr, \tag{7}$$

$$\varphi_2 = \theta + \psi(\alpha_0,\theta) - \int_r^{r_0} \frac{\operatorname{tg}\alpha(r,\varphi_2)}{r} dr .$$

The relation between the angle of incidence at the current point in the medium and the absorption coefficient is determined by a relation similar to Snell's law (1), which can be obtained from the requirement of minimum value using variational methods:

$$n(r,\varphi) r \sin\alpha(r,\varphi) + \int_r^{r_0} \frac{\partial}{\partial\varphi} \left(\frac{n(r,\varphi)}{\cos\alpha(r,\varphi)} \right) dr = p . \tag{8}$$

Thus, the system of integral equations (6)-(8) provides us a complete description of the amplitude trajectory along which the radiation propagates, making the maximum contribution to the field on the surface. As a result of solving this equation system, it is possible to obtain a direct relationship between the measured dependence $L(\psi,\theta)$ and the internal characteristics of the medium. However, the exact solution of the direct and inverse problems based on the obtained dependencies is an almost unsolvable problem. Therefore, it is necessary to consider various approximations.

IV. Tomography of slightly inhomogeneous media

Firstly, consider the solution of the inverse problem in the simplest case of a slightly inhomogeneous medium, in which $n(r,\varphi) \approx n_0$, where n_0 denotes a constant value that resembles the known value of the imaginary part of the refractive index on the surface. Physically, this means that the ray trajectory in the medium is considered to be straight, and only small changes of the absorption coefficient at each point of the path are taken into account. These changes determine the differences in the attenuation measured on the surface. In this case, the system of equations (6) - (8) becomes significantly simpler.

First of all, Snell's law (8) takes the form of $r \sin\alpha = r_0 \sin\alpha_0 = r_{\min}$, i.e. the current angle of incidence $\alpha(r)$ does not depend on the properties of the medium, but is determined only by the initial angle of incidence α_0 and the radius of the considered point r. Application of the obtained relation to equations (6) - (7) gives

$$L = \int_{r_{\min}(\alpha_0)}^{r_0} \frac{r dr}{\sqrt{r^2 - r_{\min}^2}} \left[n(r,\varphi_1) + n(r,\varphi_2) \right],$$

$$\psi = \pi - 2\alpha_0, \quad \varphi_{1,2} = \theta - \alpha_0 + \frac{\pi}{2} \mp \arccos\frac{r_0}{r} .$$

To restore the dependence $n(r,\varphi)$, it is necessary to expand the measured and investigated functions in a Fourier series in variables θ and φ_j $(j = 1,2)$ [3]:

$$L(\alpha_0,\theta) = \sum_{k=-\infty}^{\infty} L_k(\alpha_0) e^{ik\theta}, \quad n(r,\varphi_j) = \sum_{k=-\infty}^{\infty} n_k(r) e^{ik\varphi_j} \tag{9}$$

The expression for the Fourier coefficients has the form of

$$L_k(\alpha_0) e^{-ik\psi/2} = 2 \int_{r_{\min}}^{r_0} \frac{r dr}{\sqrt{r^2 - r_{\min}^2}} n_k(r) T_k\left(\frac{r_{\min}}{r} \right), \tag{10}$$

where $T_k(x) = \cos(k \arccos x)$ is the Chebyshev polynomial of the first kind. This equation can be solved in various ways. In particular, using the causal solution [4], we obtain

$$n_k(r) = -\frac{1}{\pi} \int_r^{r_0} \frac{dr_{\min}}{\sqrt{r_{\min}^2 - r^2}} T_k\left(\frac{r_{\min}}{r} \right) \frac{d}{dr_{\min}} \left(L_k(\alpha_0) e^{-ik\psi/2} \right)$$

Similarly, it is possible to write a non-causal solution or obtain it using the convolution equation method [3].

V. Tomography of a medium with consideration of the ray trajectory curvature

For solving the two-dimensional inverse problem with consideration of the ray trajectory curvature in the medium, depending on its characteristics, the most developed methods are based on the use of perturbation theory. In this case, the profile studied is presented in the form of $n(\vec{r}) = n_0(\vec{r}) + \tilde{n}(\vec{r})$, where the dependence $n_0(\vec{r})$ is considered to be known, and the value $\tilde{n}(\vec{r})$ is small compared to the first term. The smallness of the function $\tilde{n}(\vec{r})$ means that the ray trajectory is determined mainly by the influence of the profile $n_0(\vec{r})$, and the value $\tilde{n}(\vec{r})$ has only a perturbing effect on the received radiation characteristics.

Let us consider the solution of the two-dimensional tomographic problem in the application of the perturbation method to the case of axisymmetry. This means that the imaginary part of the refractive index is represented in the form of $n(r,\varphi)=n_0(r)+\tilde{n}(r,\varphi)$, where $n_0(r)$ denotes the axisymmetric component, generally unknown, $\tilde{n}(r,\varphi)$ denotes the function that determines the dependence of the profile on the angular coordinate, $\tilde{n}(r,\varphi) << n_0(r)$. The predominant influence of the axisymmetric component in the profile suggests that the curvature of the trajectory will be determined by Snell's law (1) $n_0(r) r \sin\alpha(r) = p$.

Substituting this relation into the system of equations (6)-(7), we obtain

$$L(p,\theta)=\int_{r_{\min}(p)}^{r_0} \frac{n_0(r) r dr}{\sqrt{(n_0(r) r)^2 - p^2}} [n(r,\varphi_1)+n(r,\varphi_2)],$$

$$\psi(p) = 2p \int_{r_{\min}(p)}^{r_0} \frac{dr}{r\sqrt{(n_0(r) r)^2 - p^2}}, \tag{11}$$

$$\varphi_{1,2} = \theta + \frac{\psi(p)}{2} \mp p \int_{r_{\min}(p)}^{r} \frac{dr}{r\sqrt{(n_0(r) r)^2 - p^2}}.$$

where the turning point radius $r_{\min}$ is determined from the relation $n_0(r_{\min}) r_{\min} = p$.

(11)

To solve the problem, we use the expansion (9). Note that the zero Fourier coefficient of the function $n(r,\varphi)$ resembles the axisymmetric component $n_0(r)$. Substituting the obtained expansions in (11), we find the following equation with respect to the Fourier coefficients:

$$L_k(p) e^{-ik\psi(p)/2} = \int_{r_{\min}}^{r_0} \frac{2 n_0(r) n_k(r) r dr}{\sqrt{(n_0(r) r)^2 - p^2}} \cos\left(kp \int_{r_{\min}}^{r} \frac{d\rho}{\rho\sqrt{(n_0(\rho)\rho)^2 - p^2}} \right), \tag{12}$$

When $k=0$, this expression resembles the equation (2) for an axisymmetric medium with the replacement: $L \to L_0$, $n \to n_0$, and therefore, to solve this equation, it is possible to use formulas (4), (5).

To determine the components in the case of $k \neq 0$, we introduce the following notations:

$$v = n_0(r) r \quad , \quad b = n_0(r_0) r_0 \quad , \quad \varphi_k(v) = n_k(r)\frac{dr}{dv} \quad ,$$

$f_k(p) = L_k(p) e^{-ik\psi(p)/2}$, and with that, (12) takes the form of

$$f_k(p) = \int_p^b \frac{2v dv}{\sqrt{v^2 - p^2}} \varphi_k(v) \cos\left(kp \int_p^v \frac{dw}{\sqrt{w^2 - p^2}} \frac{\varphi_0(w)}{w} \right), \tag{13}$$

where the function $\varphi_0(v)$ corresponds to the axisymmetric dependence $n_0(r)$, which is already known, and $w = n_0(\rho)\rho$. There is no exact solution to this equation, however, its form is similar to the Abel transform (for $k=0$) and to the expression (10). And this similarity determines the possibility of finding a solution based on a generalization of these integral transforms.

Since the solution of equation (13) for an arbitrary form of the function $\varphi_0(w)$ is difficult, we expand this function in a power series in the neighborhood of the point $w = p$, restricting ourselves to three terms of the expansion

$$\varphi_0(w) \approx a_0 + a_1 (w/p) + a_2 (w/p)^2,$$

then $f_k(p) =$

$$= \int_p^b \frac{2\varphi_k(v) v dv}{\sqrt{v^2 - p^2}} \cos\left(a_k \arccos\frac{p}{v} + b_k \operatorname{arch}\frac{v}{p} + c_k \sqrt{\left(\frac{v}{p}\right)^2 - 1} \right), \tag{14}$$

where $a_k = k a_0$, $b_k = k a_1$, $c_k = k a_2$.

Let us find a solution to this equation in the form of

$$\varphi_k(v) = -\frac{1}{\pi} \int_v^b \frac{f_k'(q) dq}{\sqrt{q^2 - v^2}} \operatorname{ch}\left(a_k \operatorname{arch}\frac{q}{v} + b_k \arccos\frac{v}{q} + c_k \sqrt{1 - \left(\frac{v}{q}\right)^2} \right), \tag{15}$$

To verify the validity of this assumption, let us substitute equation (15) into relation (14) and make sure that the identity is obtained. Changing the order of integration, and considering that $f_k(b) = 0$, we come to the following conclusion: the identity will be satisfied if the internal integral is reduced to the form of

$$J = \int_0^1 \frac{ds}{\sqrt{s}\sqrt{1-s}} \left[\cos\left(a_k \arccos z_0 + b_k \operatorname{arch}\frac{1}{z_0} + c_k \frac{a\sqrt{s}}{p} \right) \cdot \operatorname{ch}\left(a_k \operatorname{arch} z_1 + b_k \arccos\frac{1}{z_1} + c_k \frac{a\sqrt{1-s}}{\sqrt{p^2+a^2}} \right) \right], \tag{16}$$

with the notations of $z_0 = \dfrac{p}{\sqrt{p^2 + s a^2}}$, $z_1 = \dfrac{q}{\sqrt{p^2 + s a^2}}$, $a^2 = q^2 - p^2$ for any values of parameters a_k, b_k, c_k, p, q will be equal to π. Note that this statement was proved in [5, 6] in the case when $b_k = 0$. According to the residue theory [7], this integral can be transformed into $J = \pi C$, where C denotes the value of the expression in square brackets for (16) at an infinitely remote point.

Firstly, consider the case of $a_k \neq 0, b_k \neq 0, c_k = 0$. Let us denote the expression in square brackets in (16) as $g(s)$. To determine the behavior of this function in the case of $s \to \infty$, we use the representations of the inverse trigonometric and hyperbolic functions through the logarithmic function, obtaining

$$g\big|_{s\to\infty} = \cos^2\left(b_k \ln\sqrt{s}\right). \tag{17}$$

The resulting asymptotic estimate does not depend on the value of the parameter a_k and for $b_k = 0$ the constant is $C = 1$.

For $b_k \neq 0$ and $s \to \infty$, the cosine value is not defined. To find the limit value of the function g, let us differentiate (17) with respect to the parameter b_k:

$$\left.\frac{\partial g}{\partial b_k}\right|_{s\to\infty} = -2\cos\left(b_k \ln\sqrt{s}\right)\sqrt{1-\cos^2\left(b_k \ln\sqrt{s}\right)}\left(\ln\sqrt{s}\right).$$

For boundedness of the derivative in the case of $s \to \infty$, it is necessary to require the vanishing of one of the factors for arbitrary b_k. This leads to the requirement of $\cos\left(b_k \ln\sqrt{s}\right) = 1$ for any value of the parameter b_k. Thus, we obtain that $C = 1$ for arbitrary values of a_k and b_k.

Similarly, it is possible to prove that $C = 1$ in the case when all three coefficients a_k, b_k, c_k are not equal to zero. Note that when $s \to \infty$ the last terms in the arguments of the trigonometric and hyperbolic functions in (16) become dominant, therefore, the values of the parameters a_k and b_k do not affect the behavior of the function g at an infinitely distant point. Thus, it is necessary to find the limit in the case of $s \to \infty$ for the function

$$g = \cos\left(c_k \frac{a\sqrt{s}}{p}\right)\cdot\cos\left(c_k \frac{a\sqrt{s}}{\sqrt{p^2+a^2}}\right).$$

To do this, we differentiate this function by the parameter c_k:

$$\left.\frac{\partial g}{\partial c_k}\right|_{s\to\infty} = -a\sqrt{s}\left[\frac{1}{p}\cos\left(c_k\frac{a\sqrt{s}}{q}\right)\sqrt{1-\cos^2\left(c_k\frac{a\sqrt{s}}{p}\right)} + \right.$$
$$\left. + \frac{1}{q}\cos\left(c_k\frac{a\sqrt{s}}{p}\right)\sqrt{1-\cos^2\left(c_k\frac{a\sqrt{s}}{q}\right)}\right].$$

According to the requirement of continuity of the function g, similarly to the case considered earlier, it is acknowledged that for any value of the parameter c_k the condition $\cos\left(c_k \frac{a\sqrt{s}}{p}\right) = \cos\left(c_k \frac{a\sqrt{s}}{q}\right) = 1$ must be satisfied. Consequently, the value of the constant C for arbitrary values of the parameters a_k, b_k and c_k equals to one, and the integral $J = \pi$. This fact is also confirmed by the results of numerical calculation of the integral (16). As a result, expression (15) can be used to find the functions $\varphi_k(v)$ for the case of $k \neq 0$.

To determine the desired function $n(r)$, it is necessary to consider the differential equation $\varphi_k(v) = n_k(r)\frac{dr}{dv}$. As a result of its solution, the expression for the spectral decomposition harmonics of the desired function for $k \neq 0$ can be given by

$$n_k(r) = \frac{1}{\pi}\frac{d}{dr}\int_{n_0(r)r}^{n_0(r_0)r_0} d\left(L_k(p)e^{-ik\psi(p)/2}\right)Q[p, n_0(r)r], \qquad (18)$$

where

$$Q[p, n_0(r)r] = \int_{n_0(r)r}^{p} \frac{dq}{\sqrt{p^2-q^2}}\cdot$$
$$\cdot\,\mathrm{ch}\left(a_k \operatorname{arch}\frac{p}{q} + b_k \arccos\frac{q}{p} + c_k\sqrt{1-\left(\frac{q}{p}\right)^2}\right).$$

The dependence $n_0(r)$ used in (18) can be found on the basis of the equation (5) solution. The final reconstruction of the dependence of the medium refractive index imaginary part on spatial coordinates $n(r, \varphi)$ is obtained by summing the Fourier series.

Thus, the two-dimensional inverse problem solution of an inhomogeneous absorbing medium transmission was obtained in the approximation of sufficiently smoothly changing characteristics. As a result, the radiation propagation path in the medium can be effectively approximated by Snell's axisymmetric law. The advantage of this approach is that the solution is expressed in a closed form, thus facilitating the problem regularization, and also making it possible to further refine the structure of the medium using various iterative methods. In the process of solving the problem, a new Abel type integral transform was obtained. Researching the possibility of solving problems in this and other directions, combined with the further development of mathematical methods based on the development of the Abel type transformations, will provide a powerful and promising tool for diagnosing and restoring the internal structure of various structures.

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